Microwave Measurement of Surface Resistance by the Parallel-Plate Dielectric Resonator Method

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***Abstract*—Analysis and experimental results are presented for a parallel-plate dielectric resonator method to measure the surface resistance of conducting or superconducting plates. In the present paper, three main questions are considered in detail: the influence of the relative sizes of the conducting or superconducting plates on the measured value of the surface resistance R_s, the influence of the shape of the plates on the R_s measurement, and how to interpret obtained results. Measurements were made at resonant frequencies of 14.1–14.5 GHz in a temperature range between 77 and 300 K.**

I. Introduction

Dielectric resonators (DR's) are often used in microwave circuits. The DR's have brought significant improvement in the design of microwave oscillators and filters. The DR's with low loss-factor can be used for microwave measurement of the surface resistance of conducting or superconducting materials. The method described in the present paper is one of the numerous resonator configurations in which the stored energy outside the dielectric is negligible and, hence, leads to a high filling factor. In the parallel-plate dielectric resonator (PPDR) method or, also well known as Courtney method [1], a dielectric rod resonator is short-circuited at both ends by two parallel conducting or superconducting plates. The PPDR is one of the most simple and suitable configuration for microwave measurement of the surface resistance, particularly, for high-temperature superconductors (HTS) due to the fact that the PPDR method does not require attaching any contacts, patterning, or etching. A configuration of the PPDR used in measurements is shown in Fig. 1. A cylindrical dielectric rod resonator of diameter $2R$ and length L is placed between two parallel conducting plates of diameter $2r$. The resonant frequency of the TE_{011} mode and Q-factor are measured in the transmission method. The EM field of the resonator is excited and detected by two semi-rigid coaxial cables, each having a small loop at the end. The TE_{011} mode is excited by horizontally orienting the loop. The coupling can be adjusted by changing the insertion depth of the cables. For the TE_{011} mode in a cylindrical rod, the EM fields decay almost exponentially along the radial direction away from the surface of the rod. If the diameter of the plates is infinitely large, there is no radial rf leakage. In case of finite conducting plates, there is some radiation loss.

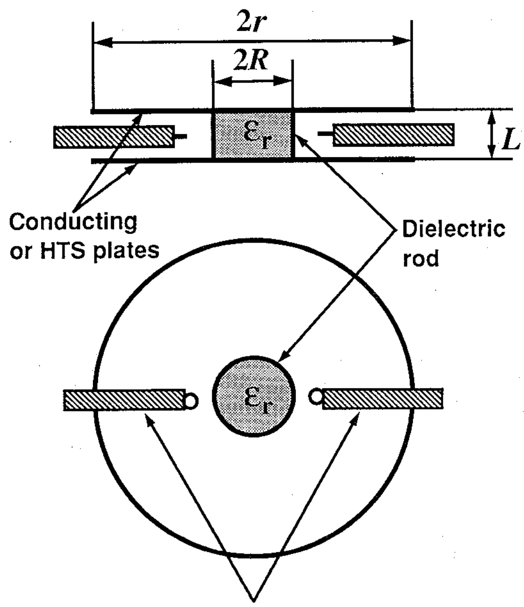

Fig. 1. Configuration of the parallel-plate dielectric resonator.

The method described here has been analyzed earlier by Courtney [1], and Kobayashi and Katoh [3] for measuring the surface resistance R_s of conducting plates and by Shen *et al.* [4] and Kobayashi *et al.* [5] for HTS materials. The same resonator configuration is considered in this paper. Here, we examine questions that have not been studied in literature [1]–[5] before. As in each method for surface resistance measurement, the PPDR has some advantages and some deficiencies. Normally, the size and shape of the HTS film is restricted by the available low-loss wafer. There is always the question of the influence of the finite size of the HTS film on the accuracy of the microwave surface resistance measurement. Kobayashi and Katoh have analyzed [3] this influence on measurements of dielectric properties of the DR itself. Shen *et al.* [4] have considered this question for a specified unloaded Q-value of the PPDR. In this paper, the influence of the finite size and shape of the conducting or superconducting plates on the surface resistance measurement will be considered. Often, the homogeneity of HTS materials is not guaranteed. What part of the heterogeneous HTS film makes more contribution to superconductor loss in the PPDR method? In other words, how

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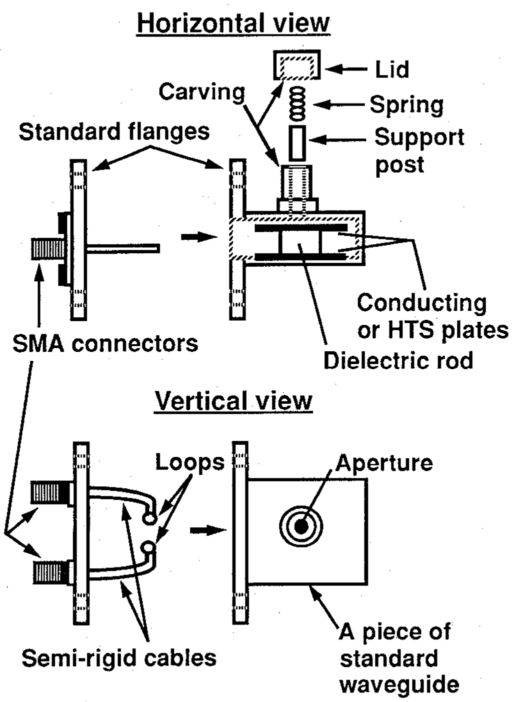

Fig. 2. Expanded view of the measurement setup configuration. The dielectric rod resonator, short-circuited at both ends by two parallel conductor or HTS plates, is held by a support post and spring. Finally, two parts of test setups are connected by four screws.

do we interpret obtained results for superconducting films? This question will be also considered in the present paper.

II. Test Setup and Dielectric Resonator Configuration

The measurements of the temperature dependence of the resonant frequency and unloaded Q-factor of the resonator were made in a liquid nitrogen dewar. The resonator is located in a shielded enclosure. A piece of 26-mm-long X-band standard waveguide, closed at one end, has been taken as a metal package. A full test setup is shown in Fig. 2. At another end of the waveguide, a standard flange is used for closing the cavity. The test chamber is silver plated inside. Two standard SMA connectors are fixed on the package (input and output). Two pieces of semi-rigid cables with small loops at the end are used for coupling. The resonator is located in the middle of the package and held by support post and spring. The dielectric support post is made from Teflon®. The temperature is measured with a iron-constantan thermocouple and a HP3478A voltmeter. The microwave measurements are performed with the aid of a HP8510B vector network analyzer. The connection between network analyzer and test setup is realized by two standard semi-rigid 40-cm-long cables. HP3478A voltmeter and HP8510B vector network analyzer are connected to a HP computer. All measurements are controlled by computer and are realized "on the fly" or point by point. The transmission coefficient S_{21} and the loaded Q-value, Q_L, of the resonators are measured with the well known 3-dB bandwidth method. The values of the transmission coefficients S_{21} and S_{12} are the same within the tolerance (1–2%).

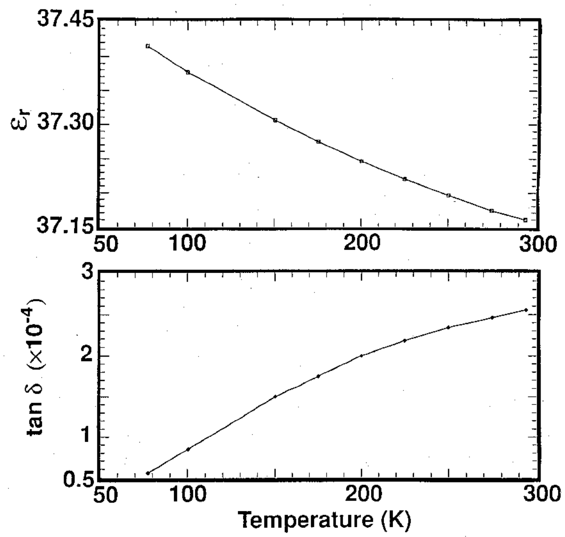

Fig. 3. Measured temperature dependences of the relative dielectric constant and loss-tangent of the DR at 14.5 GHz.

The measurements were performed with the dielectric resonator made from (Zr, Sn)TiO_4, which has a radius $R = 2.74 \pm 0.004$ mm and a length $L = 2.164 \pm 0.004$ mm (Siemens AG). In the PPDR configuration at room temperature, the resonant frequency of the TE_{011} mode is equal to 14.501 GHz. At this frequency and at room temperature, the values of the dielectric constant and the loss factor are $\varepsilon_r = 37.16 \pm 0.093$ and $\tan\delta = 2.56 \pm 0.028 \times 10^{-4}$, respectively. The measured temperature dependences of ε_r and $\tan\delta$ are shown in Fig. 3. In the temperature range of 77–300 K, the frequency coefficient and the dielectric constant coefficient are $\tau_f = 9.7$ ppm/K and $\tau_\varepsilon = -31.4$ ppm/K, respectively. The coefficient of thermal linear expansion is assumed to be $\tau_L = 6$ ppm/K. The DR consists of 79% of Zr and 21% of Sn. The procedure of the measurement of the dielectric properties is completely described in [7].

III. Theory

The unloaded Q-value, Q_u, of a resonator is defined [2] as

$$\frac{1}{Q_u} = \frac{1}{Q_c} + \frac{1}{Q_d} + \frac{1}{Q_r} \tag{1}$$

where Q_c is conductor Q, Q_d is dielectric Q and Q_r is radiation Q. Each Q-value, Q_i, where i can be c, d or r, is defined [2] as

$$Q_i = \frac{2\pi f_r W_0}{P_i} \tag{2}$$

where P_i is the dissipated power in the conductor, when $i = c$, in the dielectric, when $i = d$ and P_r is the radiated power at the resonant frequency f_r; W_0 is the total stored energy in the system. At the resonant frequency, W_0 is equal to the peak stored electrical energy and, consequently, can be presented as

$$W_0 = W_{E1} + W_{E2} \tag{3}$$

where W_{E1} and W_{E2} are the peak electrical energy stored inside and outside the dielectric rod at the resonant frequency, respectively. By using the definition of the dielectric loss tangent, $\tan\,\delta$:

$$\tan\,\delta = \frac{P_d}{2\pi f_r W_{E1}} \tag{4}$$

the inverted dielectric Q-factor can be expressed as

$$\begin{aligned}\frac{1}{Q_d} &= \frac{P_d}{2\pi f_r W_0} = \frac{W_{E1}}{W_0}\tan\,\delta \\ &= \frac{W_{E1}}{W_{E1}+W_{E2}}\tan\,\delta = \frac{\tan\,\delta}{1+w}\end{aligned} \tag{5}$$

where $w = W_{E2}/W_{E1}$ is the ratio of the stored electrical energy outside to the stored electrical energy inside the rod.

The power loss in conducting or superconducting plates can be approximated [2] by

$$P_c = \frac{1}{2}\int_S R_s|\vec{J}_s|^2 dS = \tfrac{1}{2}R_s \int_S |\vec{H}|^2 dS \tag{6}$$

where R_s is the surface resistance of conducting or superconducting plates; J_s and H are the current and magnetic field, respectively, for perfectly conducting plates. This approximation is valid for good conductors at microwave frequencies [2]. Here, we assume that R_s is constant on the surfaces of the plates but a function of temperature.

The expression for R_s can be obtained from (1) by taking into account that in the PPDR configuration there are two conducting plates and by substituting (2), (5) and (6) into (1):

$$R_s = \frac{2\pi f_r W_0}{\displaystyle\int_S |\vec{H}|^2 dS}\left(\frac{1}{Q_u} - \frac{\tan\,\delta}{1+w} - \frac{P_r}{2\pi f_r W_0}\right). \tag{7}$$

The values of W_0, P_r and w in (7) can be determined by knowing the distribution of the EM fields for the $TE_0 11$ mode. The exact expressions for the EM field components can be derived in case of infinite conducting plates, $r = \infty$. The PPDR can be treated as a section of circular dielectric waveguide short-circuited by the conducting or superconducting plates at both ends. The TE_{011} mode EM fields between two plates can be directly derived from the TE_{01} traveling wave fields in a dielectric waveguide [2], [4]. For simplicity, the losses in the DR and conducting or superconducting plates can be neglected in the field analysis, the loss effects can be considered as a perturbation. The nonzero field components in a cylindric coordinate (ρ, φ, z) system, where axis z coincides with the axis of the DR, are $E_\varphi(\rho, z), H_\rho(\rho, z)$ and $H_z(\rho, z)$. The exact expressions for the EM field components, the total stored energy in the resonator W_0, and the ratio of the stored electrical energy outside to the stored electrical energy inside the rod w are described elsewhere [1]–[5].

In case of infinite size plates ($P_r = 0$), by using expressions for the EM fields and the total stored energy W_0, (7) can be presented as

$$R_s = \frac{240\pi^2\varepsilon_r}{1+\varepsilon_r w}\left(\frac{L}{\lambda}\right)^3\left(\frac{1+w}{Q_u} - \tan\,\delta\right) \tag{8}$$

where ε_r is the relative permittivity and λ is a wavelength. The values of loss-factor $\tan\,\delta$ and the relative permittivity ε_r of the DR have to be known before performing R_s measurement. The values of f_r and Q_u have to be measured.

From the resonance curve on a network analyzer, loaded Q_L and unloaded Q_u values are determined [2] by

$$Q_L = \frac{f_r}{f_2 - f_1} \tag{9}$$

$$Q_u = \frac{Q_L}{1 - a_0} \tag{10}$$

$$a_0 = 10^{-IL(dB)/20} \tag{11}$$

where $f_2 - f_1$ is the half-power bandwidth, and IL is the insertion loss at $f = f_r$ in dB.

IV. Measured Results

The expression (8) for the surface resistance R_s is derived in case of the infinite conducting or superconducting plates ($r = \infty$). In case of finite plates, the measured resonant frequency f_r of the TE_{011} mode and the measured unloaded Q-factor will be slightly different. Consequently, the obtained value of R_s at given temperature has to be corrected.

The dependencies of $f_r(r, T)$ and $Q_u(r, T)$, where r is the radius of the plates and T is the temperature, were measured. The temperature range 77–300 K is taken for our measurements. Round copper conducting plates with the different size in radius are used in the experiment. They were polished for each measurement. The radius of the DR is $R =$ 2.74 mm. A set of the copper plates with $r_1 = 2.74$ mm, $r_2 = 3.01$ mm, $r_3 = 3.425$ mm, $r_4 = 4.115$ mm, $r_5 =$ 5.48 mm, $r_6 = 8.22$ mm, and $r_7 = 11.25$ mm is tested. The dimension of the minimum size plates, r_1, is determined by the radius R of the DR. It makes no sense to perform measurements with the size of the plates less than the diameter of the DR. The dimension of the maximum size of the plates, r_7 is determined by the internal size of the test setup. In terms of the DR radius, R, the radii of the plates are equal to $1R; 1.1R; 1.25R; 1.5R; 2R; 3R$, and $4.1R$, respectively.

The measured dependencies of the resonant frequency and the unloaded Q-factor at different temperatures are presented in Figs. 4 and 5. The measured values of f_r and Q_u for the plates with $r_6 = 8.22$ mm $= 3R$ and $r_7 = 11.25$ mm $= 4.1R$ are the same. In Figs. 4 and 5 we do not present $r_7 = 4.1R$, in order to keep the graphics more legible. The difference between measured $Q_u(r, T)$ values with $r_5 = 2R$ and $r_6 = 3R$ is practically negligible; at low temperatures there is a small difference between these two values. The value of the resonant frequency is less affected by decreasing the size of the conducting plates than the unloaded Q-factor. Even at $r_4 = 1.5R$ the value of the resonant frequency is equal to the value of the resonant frequency at $r_6 = 3R$.

From the measured plots we can infer that only the point $r_4 = 1.5R$ can be considered the minimum reasonably possible size of the plates for R_s measurement. The value of the resonant frequency at $r_4 = 1.5R$ is equal to the value of the resonant frequency at $r_6 = 3R$. The value of $Q_u(1.5R, T)$ ranges from 86% (at 300 K) to 90% (at 77 K) at the value of $Q_u(3R, T)$. The difference between these two values depends on the temperature. From (8) the ratio

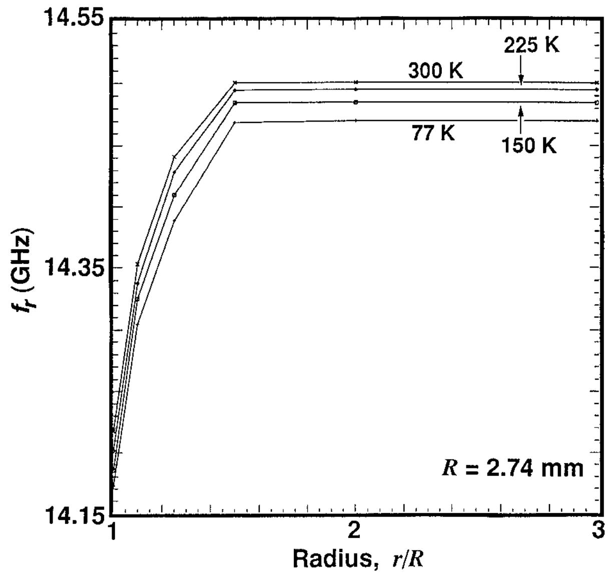

Fig. 4. Measured dependence the resonant frequency of the TE_{011} mode versus relative size of the copper plates, r/R, at different temperatures. The R is the radius of the DR.

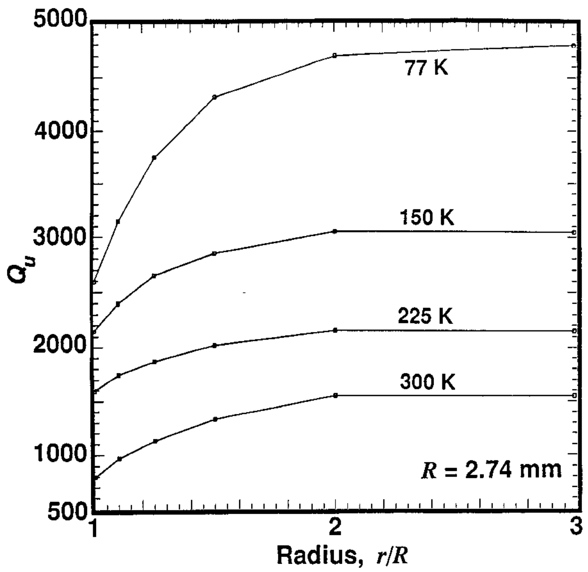

Fig. 5. Measured dependence the unloaded Q-factor versus relative size of the copper plates, r/R at different temperatures. The R is the radius of the DR.

$(R_s(1.5R,T)-R_s(3R,T))/R_s(3R,T)$ is placed in an interval between 15% (at 300 K) and 28% (at 77 K). The value of this ratio depends on the temperature. The performance Rs measurement with small size of the plates gives a value of the surface resistance with the error in a positive direction. The absolute measured values of the conductivity, $\sigma = 1/R$ with the size of the plates $r > 2R$ are placed in a range 84–89%, compared to calculated value using dc bulk conductivity [3], [5].

V. The Error of R_s Measurement

The measured results in Figs. 4 and 5 are not absolute because the test setup has some influence. Indeed, the measurement for the small plate size ($r < 2R$) with another enclosure can be slightly different. The measured results show that, practically, the plates with $r = 3R$ are sufficient for the Rs measurement in the metal package (in our configuration) at temperatures ranging between 77 K and 300 K. Let us analyze the (7) in terms of the energy distribution in the resonator. We can obtain an estimation for the relative error $\Delta R_s/R_{s0}$, as function of the radius r, where $\Delta R_s = R'_s(r) - R_{s,0}; R'_s(r)$ is the value of the surface resistance obtained with the finite size plates, $r; R_{s,0}$ is the value of the surface resistance obtained with the infinite size plates. The physical meaning of this error is what influence of the finite size plates has on the surface resistance measurement by the PPDR method if we use (8), derived for a case of infinite size plates? If we can estimate the value of the radiated power P_r in (7) as a function of the radius of the plates, r, the relative error $\Delta R_s/R_{s,0}$ as function of r can be obtained.

In case of the infinite conducting plates or very large size plates (in our case, $3R \le r$), when there is no radiation loss ($P_r = 0$), the value of the surface resistance $R_{s,0}$ is calculated from (8). For R_s measurement with finite conducting plates and small radiation loss $P_r \ll 2\pi f_r W_0$, we write (7) as

$$R'_s(r) = \frac{2\pi f_r W'}{\int_S |\vec{H'}|^2 dS}\left(\frac{1}{Q'_u} - \frac{\tan\delta}{1+w'} - \frac{P_r}{2\pi f_r W'}\right) \quad (12)$$

where W' is the total stored energy in the resonator with the finite size plates; H' is the tangential magnetic field on the conductor surface; Q'_u is the measured unloaded Q-factor; w' is the ratio of the stored electrical energy outside to the stored electrical energy inside the rod; f_r is the resonant frequency of the TE_{011} mode; and S is the surface of the plates. The meaning of the apostrophe is that these values are measured or calculated in case of the finite size plates. The corresponding values for a case of infinite size plates are W_0, H, Q_u, and w. In (12) we suggest that the resonant frequencies in both cases are equal. That is a good assumption for a reasonable plate size. In our case, it is valid for $r \ge 1.5R$.

The ratio of the stored electrical energy outside to the stored electrical energy inside of the rod, w, in our configuration of the DR is about 2.3 $\times 10^{-3}$. In the case of small radiation loss, the difference between w and w' (with $\varepsilon > 10$) can be considered as a second-order influence. The full stored energy W' can be estimated as $W_0 - P_r/2\pi f_r$. The value of $\int |H|^2 dS$ (multiplied by R_s) has energy units, thus, we can estimate $\int |H'|^2 dS = ((W_0 - P_r/2\pi f_r)/W_0) \int |H|^2 dS$. The value of $\Delta R_s/R_{s,0}$ after numerous substitutions can be expressed as

$$\frac{\Delta R_s}{R_{s,0}} = \frac{\frac{1}{Q'_u} - \frac{1}{Q_u} - \frac{P_r}{2\pi f_r W_0 - P_r}}{\frac{1}{Q_u} - \frac{\tan\delta}{1+w}}. \quad (13)$$

Taking into account that $P_r \ll 2\pi f_r W_0$ we rewrite (13) as

$$\frac{\Delta R_s}{R_{s,0}} = \frac{\frac{1}{Q'_u} - \frac{1}{Q_u} - \frac{P_r}{2\pi f_r W_o}}{\frac{1}{Q_u} - \frac{\tan\delta}{1+w}}. \quad (14)$$

The $P_r/2\pi f_r W_0$ value was defined in (2) as $1/Q_r$, where Q_r is the radiation Q-factor. The value of the radiation loss can be estimated as the electromagnetic energy stored in the "tail" region, $\rho > r$. If the resonator is enclosed in the metal package with good-quality walls, then the part of the EM energy stored in the "tail" is reflected back from cavity walls and is reinjected in the TE_{011} mode. The worst case is when all the "tail" EM energy is lost. We assume that the distribution of the EM fields are the same as in the case of the infinite conducting plates ($r = \infty$). Hence, the expression for $P_r/2\pi f_r W_0$ can be written as (15), shown at the bottom of the page, where J_0 and J_1 are the Bessel function of the first kind; K_0 and K_1 are the modified Bessel function of the second kind; x_1 and x_2 are ρ-direction wave numbers inside and outside the rod, respectively. The relation between ξ_1 and ξ_2 for TE_{011} mode is $(\pi/L)^2 = k^2\varepsilon_r - \xi_1^2 = k^2 + \xi_2^2$, where k is the propagation constant, $k = 2\pi/\lambda$. The first numerical factor of 1/2 on the right side of (15) (shown at the bottom of the page) derives from converting field components from peak values into average values [4]. The integrals can be easily calculated [6]. When $Q'_u = Q_u$ in (14), we receive similar estimation as Shen *et al.* in [4].

In a region $2R \leq r \leq 3R$, where Q'_u and Q_u are practically the same, the calculated relative error $\Delta R_s/R_{s,0}$, where we take $R_{s,0} = R_s(3R,T)$, is presented in Fig. 6. For a region $1.5R \leq r < 2R$, where Q'_u and Q_u are different, in order to build up the same plot, we have to know the dependence of $Q_u(r,T)$ in this region, at least, in a few points. We can only calculate this error in the point $r = 1.5R$. The calculation of the relative error $\Delta R_s/R_{s,0}$ in (14) for our configuration of the DR at point $r = 1.5R$ gives $(R_s(1.5R,T) - R_s(3R,T))/R_s(3R,T) = 14\%$ (at 300 K) to 31% (at 77 K). The value of this error depends on temperature. The calculated above result for the error in Section IV, obtained by direct calculations of R_s from (8), gives 15% (at 300 K) to 28% (at 77 K). From Fig. 6 we can infer that the calculated relative error $\Delta R_s/R_{s,0}$ strongly depends on temperature. At $r = 2R$ the ratio of values $\Delta R_s/R_{s,0}$ at 77 K and $\Delta R_s/R_{s,0}$ at 300 K is about 2.5. This is because of the temperature dependence of Q_u and $\tan\delta$. The value $P_r/2\pi f W_0$ at $r = 2R$ and 77 K is equal to 1.7×10^{-5}. Secondly, we can see from the plot a very fast decline of the curves with r, the value of $\Delta R_s/R_{s,0}$ at 77 K and at $r = 2R$ is about 10.5% and at $r = 2.5R$ is less than 0.3%. That is because of the modified Bessel functions are exponential-like.

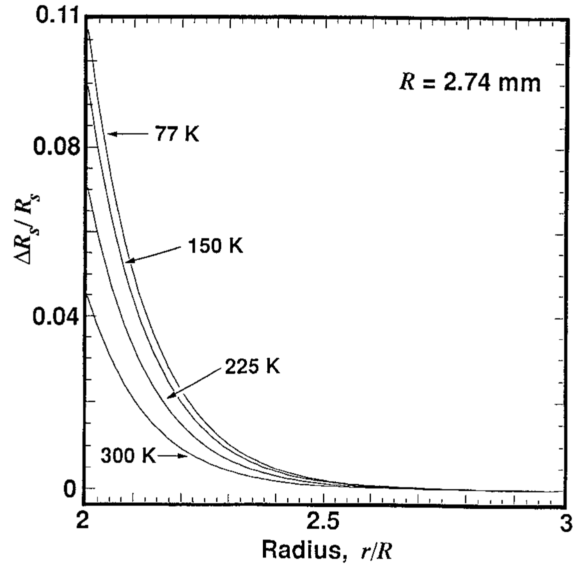

Fig. 6. Temperature dependence of the relative error $\Delta R_s/R_{s,0}$ (for more details see text) for our configuration of the DR versus relative radius of the plates, r/R. The R is the radius of the DR.

VI. Shape of Conducting or Superconducting Plates

The shape of low-loss substrates for superconducting films habitually is square. If the size of the square substrates is sufficiently large (in our configuration more than $3R$) the influence of the shape is very small and can be neglected. For the "medium" wafer size, the influence of the shape should be calculated. Often, the shape and the size of the low-loss substrate are restricted by the available HTS wafer. Hence, the question of the influence of the shape of the plates has to be considered.

The first approximation of the value of $\Delta R_s/R_{s,0}$ with square shape of the plates which have a size $2a \times 2a$ can be calculated as for round shape plates with $r = a$, as shown in Fig. 7. Let us estimate the influence of the pattern area of the square shape plates on the value of $\Delta R_s/R_{s,0}$. We calculate the effective radius $r_{\text{eff}} (a < r_{\text{eff}} < a\sqrt{2} \cong 1.414a)$ of the round shape plates that can be substituted to the square form plates with the same influence. Then we can obtain the value of $\Delta R_s/R_{s,0}$ from (14) and (15). Simple averaging of the surface, $(2a)^2 = \pi r_{\text{eff}}^2$, gives $r_{\text{eff}} = 2a/\sqrt{\pi} \cong 1.128a$ or $\Delta r = 0.128a$. But we have to take into account that each piece of the surface has to be taken with its coefficient, which, in other words, depends how far this piece of the surface is from the center of the square. The simplest way to make an estimation of the influence of the shape can be done by considering the "stored" energy outside of the resonator, or in terms of the radiated energy. Let us consider the distribution of the EM fields as it could be in case of the infinite conducting or HTS plates. We calculate the stored energy in the "tail" region for square shape of the conducting plates and then we find the r_{eff} value with equal value of the stored energy in the

$$\frac{P_r}{2\pi f_r W_0} = \frac{1}{2}\,\frac{\left[1+\left(\dfrac{\lambda}{2L}\right)^2\right]\displaystyle\int_r^\infty K_1^2(\xi_2\rho)\rho\,d\rho + \left[\dfrac{\xi^2\lambda}{2\pi}\right]^2\displaystyle\int_r^\infty K_0^2(\xi_2\rho)\rho\,d\rho}{\left(\dfrac{\xi_2 K_0(\xi_2 R)}{\xi_1 J_0(\xi_1 R)}\right)^2 \varepsilon_r \displaystyle\int_0^R J_1^2(\xi_1\rho)\rho\,d\rho + \displaystyle\int_R^\infty K_1^2(\xi_2\rho)\rho\,d\rho} \tag{15}$$

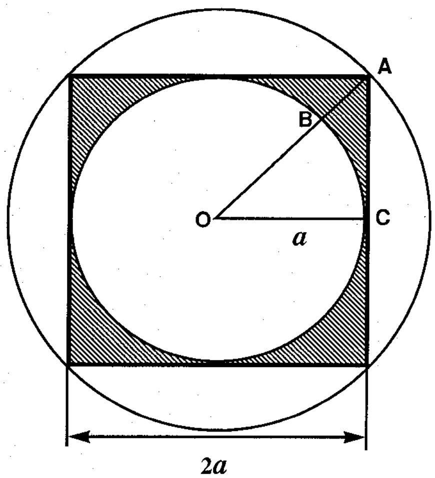

Fig. 7. Geometry of the square shape of the HTS film wafer.

"tail" region. So, we have to resolve the next integral equation

$$8\int_{S_\infty}\left(\int_0^L dW(\rho,\varphi,z)\right) = \int_0^{2\pi}\left(\int_0^L\left(\int_{r_{\text{eff}}}^{\infty} dW(\rho,\varphi,z)\right)\right) \quad (16)$$

where $W(\rho,\varphi,z)$ is an average stored EM field energy outside of the resonator, S_∞ is a surface of the AOC sector without the surface of the AOC triangle in Fig. 7, and $S_\infty = S_{\angle \mathrm{AOC}} - S_{\Delta \mathrm{AOC}}$. The method of the calculating of the value of $W(\rho,\varphi,z)$ is the same as for calculating of the value of $P_r/2\pi f W_0$ in (15) and shown elsewhere [4]. After calculating the integrals with respect to z and φ in the second integral, (16) can be rewritten as

$$8\frac{L}{2}\int_0^{\pi/4}\left(\int_{a/\cos\varphi}^{\infty} dW(\rho,\varphi)\right) = 2\pi\frac{L}{2}\int_{r_{\text{eff}}}^{\infty} dW(\rho). \quad (17)$$

After some simplifications and calculations, the first integral I_1 becomes

$$I_1 = 8\int_0^{\pi/4} d\varphi\left(\left[\left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{2L}\right)^2\right]\int_{a/\cos\varphi}^{\infty} K_1^2(\xi_2\rho)\rho d\rho + \left(\frac{\xi}{2\pi}\right)^2 \times \int_{a/\cos\varphi}^{\infty} K_0^2(\xi_2\rho)\rho d\rho\right). \quad (18)$$

After similar operations the second integral I_2 can be expressed as function of r_{eff}:

$$I_2(r_{\text{eff}}) = 2\pi\left(\left[\left(\frac{1}{\lambda}\right)^2 + \left(\frac{1}{2L}\right)^2\right]\int_{r_{\text{eff}}}^{\infty} K_1^2(\xi_2\rho)\rho d\rho + \left(\frac{\xi}{2\pi}\right)^2 \int_{r_{\text{eff}}}^{\infty} K_0^2(\xi_2\rho)\rho d\rho\right). \quad (19)$$

The integral I_1 in (18) cannot be obtained as analytical expression and have to be resolved numerically for each case. The integral I_2 in (19) can be easily calculated directly [6]. After the calculating the value of I_1, a equation $I_2(r_{\text{eff}}) = I_1$ has to be solved. Finally, the value of $\Delta R_s/R_{s,0}$ for $r = r_{\text{eff}}$ is calculated from (14) and (15).

Let us consider the most general small size of the HTS wafer, when $2a = 10$ mm for our dimensions of the DR, $R = 2.74$ mm and $L = 2.164$ mm. The first approximation for such configuration is $r = a = 5$ mm. If we assume that $Q_u = Q'_u$ in (14), then for copper plates, the calculated value of $\Delta R_s/R_{s,0}$ for $r = 1.9R = 5$ mm is equal to 9% at 300 K and 21% at 77 K. After calculating the I_1 in (18), we build up the function $I_2(r_{\text{eff}})$ graphically to obtain the value of r_{eff}. In our case we receive $r_{\text{eff}} = 1.065a = 5.325$ mm or 6.5% increasing of the radius of the plates. In terms of the additional surface that is about 4 times less than the value of the pattern surface. In terms of the radius of the DR, R we have $r = 1.94R$. From (14) and (15), where we assume that $Q_u = Q'_u$, we determine that the value of $\Delta R_s/R_{s,0}$ is about 6% at 300 K and 16% at 77 K. That means that the obtained value of R_s for copper from (8) for such configuration of the PPDR is higher on 6–16% in the temperature range 77–300 K. The estimation of the value of $\Delta R_s/R_{s,0}$ at 77 K and $r = 1.94R = 5.325$ mm for the HTS films gives about 23% for our configuration of the PPDR. The conclusion is that it is better to perform the surface resistance measurement with bigger size HTS wafers for this DR.

VII. Interpretation of Obtained Results

The current distribution J_s on the surface of the plates is symmetrical, relatively, to the z-axis and does not depend on φ and can be obtained from the tangetial magnetic field H_t on the surface conductor or superconductor surface. The exact expressions for H_t are described elsewhere [1]–[5]. We would like to mention only that inside the dielectric rod ($\rho \leq R$) the tangetial magnetic field H_t on the surface conductor or superconductor surface $H_t \propto J_1(\xi_1\rho)$, and outside the rod ($\rho \geq R$)$H_t \propto K_1(\xi_2\rho)$. Thus, we can present the current distribution in ρ-direction in normalized units, Fig. 8. The absolute value of J_s depends on the power inserted into the resonator and the power dissipated on the conducting plates, and can be calculated accurately [4], [5]. The maximum value of the rf current density on the surface of conducting or superconducting plates is at $\xi_1\rho_{\max} = 1.84118$, here $\rho_{\max}$ is the radius of a cycle where the rf current density reaches the maximum value.

The power dissipated in the conductor P_c for both plates can be obtained from (6) as

$$P_c = P_{c1} + P_{c2} = 2\frac{R_s}{2}\left[\int_{S1}|H_t|^2 dS + \int_{S2}|H_t|^2 dS\right] \quad (20)$$

where S_1 is the surfaces covered by the rod, $\rho \leq R$, and S_2 is the remaining area of the conducting plate, $\rho \geq R$. In order to determine the contribution to the conductor loss from each part of the surface, let us create next function $P_c(\rho)$, which correlate to the density of the conductor loss. By substituting the exact expressions for H_t to (20) and calculating the integrals [6], the function $P_c(\rho)$ can be written

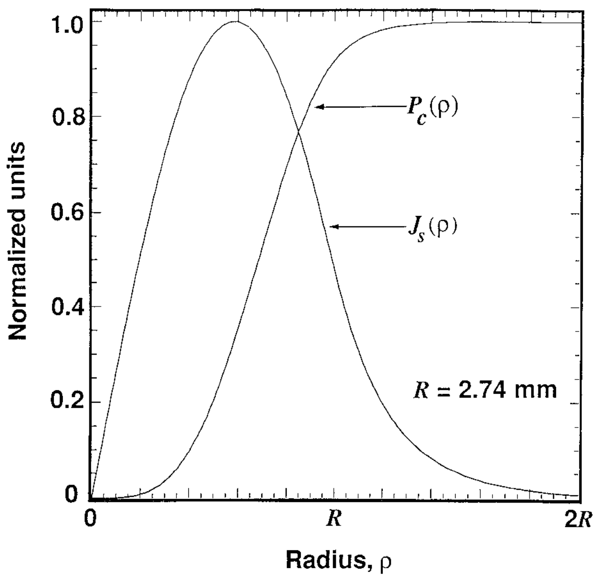

Fig. 8. Normalized current density, $J_s(\rho)/J_{s,\max}(\rho)$ and normalized density of the conductor loss, $P_c(\rho)/P_c(\infty)$ on the surface of conductor plates versus distance (radius) from the center of the DR. The R is the radius of the DR.

as

$$P_c(\rho) = \pi R_s\left(A\frac{\pi}{L}\right)^2\left(\frac{\rho}{\xi_1}\right)^2(J_1^2(\xi_1\rho) - J_0(\xi_1\rho)J_2(\xi_1\rho)) \tag{21}$$

for $\rho \leq R$, and

$$P_c(\rho) = \pi R_s\left(A\frac{\pi}{L}\right)^2(B_0R^2 + F(\rho)) \tag{22}$$

where A is the constant;

$$B_0 = \frac{J_1^2(\xi_1 R) - J_0(\xi_1 R)J_2(\xi_1 R)}{\xi_1^2} - \left(\frac{J_0(\xi_1 R)}{K_0(\xi_2 R)}\right)^2 \times \frac{K_1^2(\xi_2 R) - K_0(\xi_2 R)K_2(\xi_2 R)}{\xi_2^2} \tag{23}$$

$$F(\rho) = \left(\frac{J_0(\xi_1 R)}{K_0(\xi_2 R)}\right)^2\left(\frac{\rho}{\xi_2}\right)^2(K_1^2(\xi_2\rho) - K_0(\xi_2\rho)K_2(\xi_2\rho)) \tag{24}$$

for $\rho > R$.

By using the practical values of R, L, ε_r and f_r we can build up the function $P_c(\rho)$ graphically. The function $P_c(\rho)$ in normalized units is presented in Fig. 8. The absolute value of $P_c(\rho)$ depends on the power inserted to the resonator and can be determined accurately [4], [5]. Here we assume that the measured resonant frequency with finite size conducting plates is the same as in case of the infinite size plates. The main contribution, 71% to the conductor loss for our DR dimensions, appears from the surface of an annulus confined between two circles with radii $0.5R$ and R. The contribution from the surface within R is about 92%. The 77.5% from these 92% is contributed by the surface of the annulus confined between two cycles with radii $0.5R$ and R.

The plot of the function $P_c(\rho)$ for our configuration of the PPDR helps to understand the results of the R_s measurement. The small hole exactly in the middle of the conducting or HTS plates will not affect the measured value of R_s. If there is a scratch on the surface of the plates somewhere in the region between $0.4R$ and R, particularly, around $\rho_{\max}$, where the surface current density reaches the maximum value $J_{\max}$ (in our configuration $\rho_{\max} = 0.583R = 1.5985$ mm), it will affect many times more on the measured value of R_s than the same scratch somewhere in the middle of the DR or in the region outside $1.5R$. For the HTS materials, in case of slightly heterogeneous films, the obtained results of the R_s value, first of all, characterize the HTS film area of an annulus that lies between $0.3R$ and $1.3R$ radii, to say almost nothing about the quality of the HTS film, for example, in the center area of the DR. For pure conducting materials, this question is not important due to the homogeneity of the conducting substances.

The high values of the relative error $\Delta R_s/R_{s,0}$, obtained for small size of plates, relatively to the dimensions of the DR, should be considered as the top of the possible errors. The value of the relative error $\Delta R_s/R_{s,0}$ was derived at the condition that the radiation loss is small $P_r/2\pi f_r \ll W_0$.

Kobayashi *et al.* in [5] used the PPDR method for the surface resistance measurements of the HTS bulk plate relatively to another plate, the copper one. The conductor Q_c-factor of the copper plate is dominant in (1) at very low temperatures. When the small value is on the "top" of the high value, an attempt to subtract this small value gives a large error. If the measurements are performed with the plates made from different materials, the obtained value of R_s can be considered as a good one in case there exists no large difference of the surface resistances for both plates. In case of the plates made from the same material but with slightly different values of the surface resistance, the obtained data reflect more the highest value of R_s of one of the plates than the lowest value of R_s.

Our measurements and data obtained by Kobayashi and Katoh in [3] show that in case of copper plates the maximum measured value for a conductivity of copper $\sigma = 1/R$ at frequencies about 10 GHz is not more than 90–92% compared with that calculated using dc bulk conductivity, because of some surface roughness, oxidation, scratches etc. We can expect that the measured value of R_s by the PPDR method for the HTS films is also less on 90% than should be. On another hand, we do not need to know the value that we expect to obtain, because for practical application we have to use the measured value of R_s.

VIII. Conclusion

A theoretical analysis and experimental results are presented for a parallel-plate dielectric resonator method to measure the surface resistance of conducting or superconducting plates. Three main questions were considered in detail: the influence of the relative size of the conducting plates on the measured value of the surface resistance R_s, the influence of the shape of the plates, and how to interpret obtained recsults. Formulas for the error $\Delta R_s/R_{s,0}$ and for calculation of this error for square shape of the conducting or superconducting plates were derived and verified by experimental data with good agreement. Measurements were made at resonant frequencies of 14.1–14.5 GHz in a temperature range between 77 and 300 K.

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